Greg Kennedy Kinetics
INNOVATIVEJUGGLER.com

The Physics of Juggling

STUDY GUIDE

Can anything be fun about the study of physics? Will we ever use these things in our real lives?

If you are juggler Greg Kennedy, the answer to both of these questions is: Absolutely Yes!

Greg, whose mind for logic and science led him first to a career in engineering, has discovered the excitement and challenge of creating performance art using the fundamental concepts of physics. His shows, which are as mind-boggling as they are entertaining, illustrate the principles of motion, light, energy and, of course, gravity. But they also illustrate something else: that determination and imagination can turn an academic pursuit into a powerful art form.

Sit back and enjoy Greg's show, which includes pieces that have won him international awards and acclaim from his peers in the juggling community. And in this Study Guide, take in some of the concepts that have guided his work.

BIOGRAPHY

In an effort to change people's ideas about juggling, Greg Kennedy fuses logic and creativity to synthesize new forms of object manipulation. Greg spent several years working as a professional engineer, in addition to his juggling career. His fascination with geometry and physics led him to create ground-breaking work with original apparatus, expanding the realm of juggling. "He is a visionary who can see the possibilities in props and movement that are obscure to the masses," writes Bill Giduz, Jugglers World magazine.

Twice, Greg has entered the highest-level juggling competition, the International Jugglers Association Championships. On both occasions he received their highest honor, the Gold Medal. As one of the first jugglers to go viral, he received over 2-million views on YouTube. He was the original 'Scientist' character in Cirque Du Soleil's TOTEM, touring with them for five years. Throughout his performance career he has shared his art with millions of people in over 50 different countries.

Greg continues to receive praise and recognition from his peers for his original contributions, broadening the fields of juggling and performance art. He presently balances his performing career with a pursuit of juggling innovation, experimenting with new shapes and surfaces as well as practicing traditional skills.

PHYSICS & JUGGLING

While watching juggling, it's easy to be mesmerized by the movement. But, don't forget that juggling can be viewed as a lesson in physics. The simplest throw and catch of one ball demonstrates a multitude of concepts from Newtonian Mechanics. When we add different objects reacting together, a whole array of brief physics lessons are presented. Here is a partial list of just some of the concepts utilized in the performance:

- Gravity
- Angle of Rebound
- Elliptical Movement
- Pendulum Movement
- Centripetal Acceleration
- Gyroscopic Stability
- Light Refraction
- Spectral Dispersion

This program touches on most basic physics concepts and demonstrates the power of physical properties. And it's just fun to watch!

GRAVITY

Gravity is the force of attraction between all masses in the universe. It is especially apparent on objects near the earth's surface. Gravity causes all objects to be pulled downward to the ground (or more specifically towards the center of the earth).

EQUATION

Two bodies of matter attract each other proportionally to their masses and inversely proportional to the square of distance between them. If the mass of one body is designated as **M**, the mass of the other as **m**, the distance between them is **r**, and **G** is the universal gravitational constant, then the force of attraction between the two bodies is:

$$F = G\frac{Mm}{r^2}$$

The universal gravity equation says that as two objects are attracted and get closer together, the force increases and the acceleration between them also increases. Since the mass of the earth is so much greater than everyday objects, and the distance from the center of the earth to objects near the surface of the earth are approximately the same, differences in an object's mass and height become negligible.

Because of this, the acceleration of gravity on everyday objects here on earth is constant and predictable!

Wow, that seems really complex, no wonder it is so hard to juggle! Actually, all the math simply states the opposite. Although it may look really confusing, what all those numbers mean is that an object that is thrown acts in a predictable manner. What goes up - must come down!

How does this affect my juggling? I can predict the position, time, and speed: how they will travel in the air when I throw them and how they fall back down. This enables me to track multiple objects in the air, creating many different types of juggling patterns.

Sometimes I juggle different objects of different masses, but they still fall back down at the same rate. Do you know why?

ANGLE OF REBOUND

Rebound is the reversal in direction of an object upon encountering a barrier. The object approaches along an angle of incidence and rebounds along an angle of reflection. The angle of incidence, θ_i is the angle measured from the incoming direction of the object to a line drawn normal (90 degrees) from the surface. The angle of reflection, θ_r is the angle measured from the line drawn normal to the surface to the outgoing direction of the object.

EQUATION

The law of reflection states that the angle of reflection and angle of incidence are always equal.

$$\theta r = \theta i$$

Because of this, one can accurately predict how a ball will rebound from a surface based on the angle at which it approaches. To change where the ball will end up, you simply have to change the point which it starts.

I use these concepts in a bounce ball routine. (That I created after accidentally breaking a marble slab in half.)

It uses two rigid surfaces placed in a V-shape: each surface is 45 degrees from the ground and 90 degrees from each other. Multiple balls are bounced off the surfaces, creating different visual patterns.

I used the physics above to help me create my routine, predicting how the balls would travel upon collision with the surfaces.

An interesting aspect of this shape is that when I throw a ball straight down one foot right of center, after two bounces, it will come straight up one foot left of center. Can you predict that with the equation above?

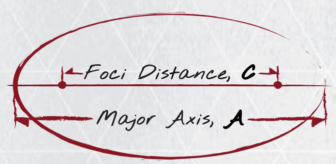
ELLIPTICAL MOTION

An ellipse is an oval. At any point on the arc, the sum of the distances to two distinct focus points is always the same. Johannes Kepler developed laws which described the motion of the planets across the sky. He showed that all planets move in elliptical orbits, with the sun at one focus.

EQUATION

Every ellipse has an eccentricity, **e** which describes how elongated it is and how far apart the two focus point are. The eccentricity of an ellipse (ranging from 0 to 1) can be defined as the ratio of the distance between the foci, **C** to the major axis of the ellipse, **A**. The eccentricity is zero for a circle and near one for an elongated ellipse

$$e = \frac{C}{A}$$



An ellipse can be approximated by rolling a ball in a bowl. Depending on the ball's initial speed and direction, the ellipse varies in eccentricity. A ball rolled quickly to one side at the rim will have a low eccentricity being very close to a circle. One rolled only slightly to the side will have a very high eccentricity and be oblong.

I experimented with rolling patterns of multiple objects on the inside of a half sphere. I was very methodical about documenting different elliptical paths a single ball could travel and then using mathematics to derive different patterns which were possible with multiple balls, not all of which I was able to learn.

I found that using the surface gave me much more control than traditional toss juggling and enabled me to perform many more objects (up to ten), but in such a small space collisions are common. Do you think it's easier to eliminate collisions with high eccentricity ellipses or low eccentricity ellipses? Why?

SIMPLE PENDULUMS

A simple pendulum is a weight suspended from a string or rod of negligible mass. It has a single resonant frequency, which means it repeats its action after a set amount of time. It will swing out, then back, then repeat itself.

EQUATION

The period, **T** is the time it takes for the pendulum to repeat itself. It can be approximated in the equation:

$$T = 2\pi\sqrt{L/g}$$

where **L** is pendulum Length, **g** is the acceleration of gravity (32.2 ft/s2), and **π** is constant (3.1415). Notice that if the pendulum length is fixed, and gravity does not change, the period does not change. This means a pendulum which is swinging high will take the same amount of time to swing out and back as one that is swinging low.

Using this idea I can accurately predict the time for a set length pendulum to return to me after releasing it. It does not matter if I release it far from the center

point or very near, it will still return after the same amount of time.

One performance piece involves manipulating pendulums from a single point. By swinging the pendulums in different paths and releasing them from different heights, I can create lots of interesting patterns. A new challenge that came along with this: was finding patterns that wouldn't tangle the strings.

The pendulums I use have 13' strings. Normally, I release a pendulum once per second; for the four pendulums, each one completes its full swing in 4 seconds. Can you see why?

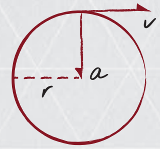
CENTRIPETAL ACCELERATION

Centripetal acceleration describes objects that travel in circular paths. Newton's laws of motion and gravitation can be used to explain the circular motion of objects in our solar system. While moving in a circular path, an object is constantly being pulled "towards the center" of the circle away from its tangential path. The pulling force constantly redirects the object forming the circular path.

EQUATION

The centripetal acceleration, **a** is proportional to the square of the velocity of the object, **v** and inversely proportional to the radius of the circle, **r**. It can be described in the following equation:

$$a = \frac{v^2}{r}$$



This can be demonstrated by rolling a ball inside a circular track.

The ball wants to travel in a straight line fulfilling Newton's laws, but the surface of the track places acceleration on the ball, causing it to travel in a circular path. For the ball to continue to travel in the track across the top of the circle, the centripetal acceleration must be greater than that of gravity.

I created a piece juggling on the inside surface of a vertical track. I spent a lot of time figuring out how to make the balls travel at a fast enough speed to stay in the track–and not too fast that I would lose control of them.

The ball stays in contact with the track until the velocity is low enough that the acceleration of the ball is lower than the acceleration of gravity; then the balls fall out so that I can catch them. If the ball goes fast enough, it can continue around the circle multiple times before I catch it.

The track I use has a 2½ ft radius. A ball must have a minimum speed of 9 ft/sec to travel across the top of the circle. Can you predict that with the equation above?

GYROSCOPIC STABILITY

A gyroscope is a rotating object suspended in a supporting structure which isolates it from outside forces. At high speeds, the gyroscope exhibits extraordinary stability of balance and maintains its direction.

A gyroscope works because of conservation of angular momentum. The angular momentum of an isolated system remains constant in both magnitude and direction. What is angular momentum? The measure of the motion of a rotating mass.

EQUATION

The angular momentum, **L** is defined as the product of the moment of inertia, **I** (the resistance of a physical object to angular acceleration) and the angular velocity, **ω** (how fast it's spinning). Below is an equation that describes this.

$$L = I \times \omega$$

If the angular velocity of an object increases (with a constant moment of inertia), the angular momentum will increase and thereby increase its stability.

The diabolo is a gyroscopic toy that was invented in China thousands of years ago. It uses a string on sticks to support a spool that can be thrown and caught.

Diabolo was one of the first juggling techniques I learned, over 25 years ago.

The stability of the diabolo allows for a great range of movement, as long as you keep it spinning. Some tricks speed the spool up. The faster it spins, the more stable it is. This also makes it easier to do tricks with.

A gyroscope has two parts: a rotating object and a supporting structure. A diabolo also has two parts: The spool and a string on sticks. Do you see the similarities?

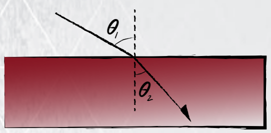
LIGHT REFRACTION

Light will change direction when it crosses a boundary of two different substances, such as from air into glass. Snell's law gives the relation-ship between angles of incidence and refraction based on each substance's index of refraction.

EQUATION

When rays or beams strike a surface and are refracted through the surface they obey Snell's equation:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



where n_1 is the index of refraction of the first material, n_2 is the index of refraction of the second material, θ_1 is the angle of incidence, and θ_2 is angle of refraction (both measured from a line perpendicular to the boundary surface).

A glass sphere has a curved surface boundary with the air. Light rays approaching from one direction will enter the ball at different angles of incidence depending where on the sphere the light hits. For this reason the light entering from one direction will scatter to many different directions creating a sparkling effect.

I perform a slightly different form of juggling manipulation by rolling multiple glass spheres around in the palms of my hands.

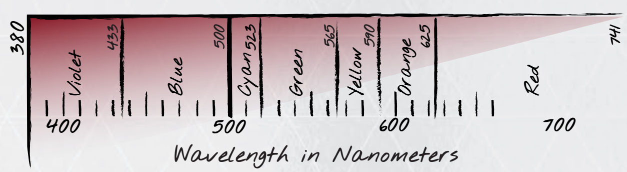
The lighting effects of this piece are almost magical. A single tiny pool of light shines on my hands. As the crystal balls slide around each other in my palms, a cascade of shimmering light is formed.

All the light comes from a single direction, yet the balls seem to be lit from within scattering light every which way. Do you know why?

SPECTRAL DISPERSION

White light includes light of all colors. It may be separated into its spectral colors by dispersion in a prism. We then see the continuous range of spectral colors (the visible spectrum). Spectral diffusion is also seen in nature in the form of a rainbow. In this case, it is water vapor in the air that causes the light to disperse.

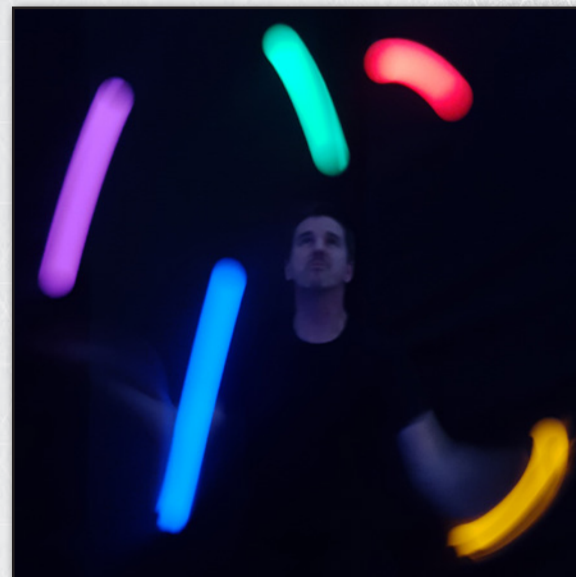
A spectral color is composed of a single wavelength and can be correlated with wavelengths as shown in the chart below.



White light also includes some types of light that are not visible to the human eye. Infrared light has a wavelength greater than 740 nm. Ultraviolet light has a wavelength below 380 nm. Neither of these can be seen without special instrumentation.

LED's and lithium ion batteries make it possible for me to illustrate spectral diffusion using juggling.

My routine starts with three glowing spheres dancing about a darkened stage swirling and spinning creating trails of light. The three finally come together forming the triangular shape of a prism. The triangle explodes into different colors: red, yellow, orange, green, blue, violet; streaking together to form a continuous rainbow.



Each of the colored illuminated balls emits light of a specific wavelength. Using the chart above, can you find the range of wavelengths for each color?

NOTE FROM ARTIST

The subject matter of this study guide may be used in conjunction with your present physics lesson plan or as a separated program to inspire future interest in physics. I feel one of the best ways to learn and generate interest is through live demonstrations of the concepts, as I provide in my program.

I hope this guide and show helps your group in this pursuit of learning. Enjoy the show!